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SLIGHTLY NONEQUILIBRIUM CONDENSATION OF A SATURATED VAPOR ON A LIQUID SURFACE

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A solution of the Barnett equations is obtained for the problem of slightly nonequilibrium vapor condensation on a liquid surface. It is shown that a Barnett sublayer with nonzero pressure gradient exists inside the nonequilibrium wall layer. The sublayer diminishes the interphase condensation resistance and induces supersaturation of the vapor.

In the present article we solve the problem of steady-state slightly nonequilibrium ("slow") condensation of a saturated vapor on a plane infinite liquid surface on the basis of the simplified Barnett equations [1]. The most complete statement of this problem is obtained within the framework of the kinetic theory of gases, where it is known to be reducible to the solution of the Boltzmann equation with the appropriate boundary condition on the surface of the condensed phase. However, under the condition of slight nonequilibrium of the condensation process the state of the vapor outside a certain Knudsen "wall layer" can be described in terms of hydrodynamical equations derived from the Boltzmann equation by the Chapman–Enskog method. The boundary conditions necessary for closing the hydrodynamical equations are deduced from the solution of the Boltzmann equation (or a modification thereof) in the Knudsen wall layer. The Navier–Stokes equations are usually used here.

In this setting, boundary conditions have been obtained for mass flux toward the surface (of the Hertz–Knudsen type) and for a temperature jump [3-7]. The temperature distribution in a nonequilibrium vapor layer has also been found [3] on the basis of the Navier–Stokes equations.

Below we formulate the slightly nonequilibrium condensation problem on the basis of the simplified Barnett distribution function obtained in [1] by a modified Chapman–Enskog method. From this function and the conditions of mass, energy, and normal momentum balance inside the vapor volume we use the standard procedure to obtain the Barnett equations, in which the one-dimensionality of the vapor flow is automatically taken into account. From the simplified Barnett function and these same balance conditions on the surface of the condensed phase we deduce boundary conditions by the classical method of Maxwell [9], closing the Barnett equations.

The stated problem is solved correct to first-order terms in the difference between the vapor pressures at infinity and on the liquid surface. We perform numerical calculations for the case of steam.

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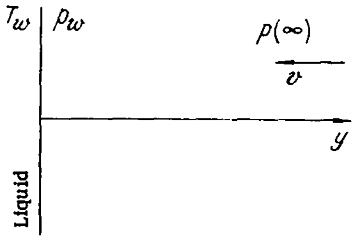

Fig. 1. Coordinate system.

Let us consider the steady-state one-dimensional condensation of a saturated vapor on a liquid surface oriented along the axis 0x. Inasmuch as the problem does not have a characteristic linear dimension, the coordinate normal to the interphase surface is referred to the mean free path λ_∞ of the molecules at a sufficient distance from the liquid phase, and the dimensionless coordinate is then denoted by y (Fig. 1). The mass flow rate, temperature, density, and pressure of the vapor are also reduced to dimensionless form (denoted by, respectively, v, T, n, and p) by means of the corresponding scales c_∞, T_∞, n_∞, and $mn_\infty c_\infty = 2p_\infty$, where c_∞ is the average velocity of the molecules, T_∞, n_∞, and p_∞ are the temperature, density, and pressure at a large distance from the surface, and m is the molecular mass. The vapor is assumed to consist of Maxwellian molecules.

Under these assumptions the Barnett equations [1] take the form

$$(pV)' = 0,$$

$$pVT' = \frac{2}{5} TVp' + \frac{3}{8} b_1 T'', \tag{1}$$

$$2pV(TV)' = -p' + \frac{2}{3} b_1 (TV)'' + \left\{ \frac{b_1^2}{6} \left[\frac{T}{p} (\ln p)'' - \frac{3}{2} \frac{T''}{p} \right]' \right\},$$

where the prime denotes differentiation with respect to the new independent variable

$$\eta = \int_0^y \frac{dy}{T} \tag{2}$$

and V = v/T is the new unknown function.

It is readily seen that Eq. (1_1) is the equation of mass conservation, (1_2) is the equation of energy conservation, and (1_3) is the energy of normal momentum conservation. Barnett terms occur only in (1_3), where they are grouped in braces for clarity. As a result of these terms the order of the system of ordinary differential equations (1) is twice the order of the corresponding Navier–Stokes system. The constant b_1 corresponds to the adopted model of Maxwellian molecules and is equal to $2/(3\pi A_2(5))$ ($A_2(5)$ is given in [2]).

Next it is required to specify the necessary number of boundary conditions describing the state of the vapor both at a large distance from the phase interface ($\eta \to \infty$) and close to it ($\eta = 0$). By assumption, as $\eta \to \infty$ the vapor is saturated and has the corresponding dimensionless temperature and pressure (density)

$$T(\infty) = 1, \; p(\infty) = 1/2 \; (n(\infty) = 1). \tag{3}$$

If we denote by $p_s(T)$ the saturated vapor pressure at the temperature T, then $p(\infty) = p_s[T(\infty)]$. We use conditions (3) as the boundary conditions for the system (1) in the case $\eta \to \infty$. We add to it the condition of smooth matching of the pressure with the unperturbed pressure $p(\infty)$

$$p'(\infty) = 0. \tag{4}$$

Note that the condition for smooth matching of the temperature $T'(\infty) = 0$ is a consequence of the first condition (3) and the form of Eqs. (1).

We obtain the boundary conditions at the phase interface by means of the simplified Barnett function f from [1]. Suppose that the surface has a temperature $T_W < 1$, which corresponds to a saturated vapor pressure $p_W = p_S(T_W) < p(\infty)$. At the interface phase $\eta = 0$ we demand that the function f satisfy the same conditions of mass, energy, and normal momentum balance as those from which the Barnett equations (1) were obtained. Following the classical Maxwell method [9], we write these conditions in the integral form

$$\int\limits_{c_y>0} Q_j c_y f(c_x,\ c_y,\ c_z)\, d\bar{c} = (1-\beta) \int\limits_{c_y>0} Q_j c_y f(c_x, -c_y,\ c_z)\, d\bar{c} + \beta \frac{2p_w}{\pi^{3/2} T_w^{5/2}} \int\limits_{c_y>0} Q_j c_y \exp(-c^2/T_w)\, d\bar{c}, \tag{5}$$

where $Q_j = 1,\ c_y,\ (1/2)c^2\ (j = 1,\ 2,\ 3)$, $f(c_x,\ c_y,\ c_z)$ is the Barnett function at $\eta = 0$, and β is the coefficient of condensation.

In the derivation of conditions (5) we use the conventional scheme of the mechanism of molecular interaction at the surface of the condensed phase, so that the coefficient of condensation (evaporation) represents the fraction of molecules captured by the surface. The flow of reflected molecules is assumed to consist of specularly [first term on the right-hand side of (5)] and diffusely (second term) reflected molecules.

Inasmuch as the Barnett function is expressed in terms of the hydrodynamic parameters and their derivatives in the Chapman–Enskog method, conditions (5) interrelate the mass flow rate, temperature, pressure, and their derivatives at the surface of the condensed phase. Essentially, on the basis of (5) for j = 1 we have a condition of the Hertz–Knudsen type, for j = 2 a temperature-jump condition at the surface, and for j = 3 another condition similar in form to the preceding one. To avoid unwieldy notation we give the specific form of conditions (5) below and then take advantage of the possibility of simplifying them to some extent. The Barnett function needed in order to compute the integrals in (5) can be found from the solution of the integral equations [1] by expension in Sonine polynomial series. The complete Barnett function [11] can also be used.

Below we give the solution of the Barnett equations (1) subject to the boundary conditions (3)-(5), assuming a slightly nonequilibrium condensation process, i.e., $\Delta p = p(\infty) - p_W \sim T(\infty) - T_W \ll 1$. As a result, the condensation is also a "slow" process: $V \ll 1$.

For completeness we first discuss the Navier–Stokes approximation of the slow condensation problem. It is obtained from the Barnett equations (1) by replacing Eq. (1_3) therein by the condition (4) of constant pressure across the nonequilibrium vapor layer. In this case Eqs. (1_1) and (1_2) are integrated in an obvious way:

$$2pV = M, \tag{6}$$

$$T = 1 + [T(0) - 1] \exp\left(\frac{4}{3} \frac{M}{b_1} \eta\right), \tag{7}$$

where the constants of integration $M < 0$ and $T(0)$ have the significance of, respectively, the mass flux and temperature at the interface. Inasmuch as the first boundary condition (3) has already been used in (7), these constants must be determined from the boundary conditions (5). In the set of conditions (5) it is customary to use the conditions for j = 1 and j = 2, written correct to the Navier–Stokes terms. In particular, from these conditions we obtain the estimates $M \sim \Delta p$, $T(0) = T_W + \tau(0)$, where the temperature jump $\tau(0)$ is also of order Δp. It is well known that the third condition (5) cannot be satisfied, due to the incompatibility of the boundary conditions (5) in application to the Navier–Stokes equations.

Consequently, assumption (4) is inapplicable to the Barnett equations as well, implying the presence of of a Barnett wall sublayer with nonzero pressure gradient directed toward the liquid phase. This effect has been noted earlier [10]. It is reasonable to assume that expression (7) retains its form in transition across the boundary of the sublayer, i.e., $T = T_W + \tau(\eta)$ and $\tau' \sim \Delta p^2$. In determining the increment to the unperturbed pressure $p(\infty)$ we stop with first-order terms in Δp, i.e.,

$$p = p_\infty + P(\eta), \tag{8}$$

where $P' \sim \Delta p$.

Under the given assumptions we obtain the following estimate from the energy equation: $\tau'' \sim \Delta p^2$. This implies that the solution for the temperature in the form (7) can be used up to the surface, including the wall sublayer, if the temperature jump $\tau(0)$ and the mass flux M are refined in accordance with the complete system of boundary conditions (5).

Retaining only terms of order Δp in (1_3), we obtain a linear third-order differential equation for the perturbed pressure P:

$$P''' - \frac{6}{b_1^2} \frac{p^2(\infty)}{T_w} P' = 0. \tag{9}$$

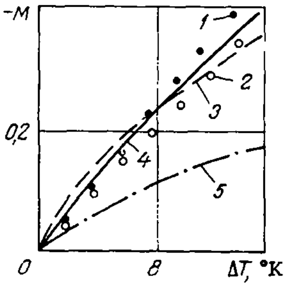

Fig. 2

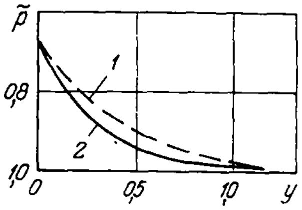

Fig. 3

Fig. 2. Mass flux M vs ΔT. 1) Navier–Stokes approximation; 2) Barnett approximation; 3) 13-time approximation; 4) BGK approximation; 5) Hertz–Knudsen formula.

Fig. 3. Normalized-pressure profiles p̃ in Barnett sublayer. 1) ΔT = 2°K; 2) ΔT = 10°K.

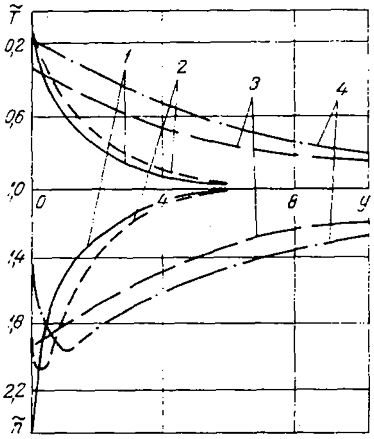

Fig. 4. Normalized temperature T̃ and density ñ. Barnett approximation: 2) ΔT = 10°K; 4) ΔT = 2°K. Navier–Stokes approximation: 1) ΔT = 10°K; 3) ΔT = 2°K.

From the physical point of view of Eq. (9) expresses the requirement that inside the Barnett wall sublayer the principal terms in the Navier–Stokes and Barnett approximations have the same order with respect to Δp. If we denote by L_B the dimensionless thickness of the wall sublayer, we obtain the following estimate from (9): $L_B \sim 1$. We know that the thickness of the main nonequilibrium vapor layer $L_m \sim \Delta p^{-1}$. Consequently, in the primary physical variables the Barnett sublayer has a thickness of the order of the molecular mean free path, whereas the thickness of the main wall layer is much greater than λ_∞ ($\Delta p \ll 1$). Using the second boundary condition (3) and condition (4), we write the solution of Eq. (9) in the form

$$P = P(0)\exp\left(-\frac{\sqrt{6}}{b_1}\,\frac{p(\infty)}{T_w}\,\eta\right), \tag{10}$$

where P(0) is a constant of integration and has the significance of the pressure at the phase interface.

After determining the integration constants M, T(0), and P(0) from Eqs. (6)-(8), (10) we obtain the final solution of the given problem. These constants must be evaluated from the boundary conditions at the surface (5). Introducing simplifications that are admissible within the framework of the Barnett sublayer, we reduce conditions (5) to the form ($\eta = 0$)

$$M=\frac{\beta}{1-0.5\,\beta}\,\frac{1}{\sqrt{\pi T_w}}\left[-\Delta p-\frac{1}{2}\left(P-\frac{1}{2}\,\frac{\tau}{T_w}\right)\right],$$

$$M=\frac{\beta}{1-0.5\beta}\,\frac{4}{5}\,\frac{1}{\sqrt{\pi T_w}}\left[-\Delta p-\frac{1}{4}\left(P+\frac{\tau}{T_w}\right)\right],\tag{11}$$

$$M=\frac{\beta}{1-0.5\beta}\,\frac{1}{4}\sqrt{\frac{\pi}{T_w}}\,\Delta p.$$

The physical meaning of the boundary conditions (11) was discussed in the derivation of conditions (5). From the mathematical point of view (11) represents a system of two linear algebraic equations for the determination of $\tau(0)$ and P(0) (M is determined automatically), the solution of which yields

$$P(0)=-\frac{32-9\pi}{12}\,\Delta p,$$

$$\tau(0)=\frac{3\pi-8}{6}\,T_w\Delta p.\tag{12}$$

Note that by putting $P\equiv 0$ in (11_1) and (11_2) we obtain the Navier–Stokes approximation for the condensation problem (with a temperature jump). We use this approximation below for comparison with the solution of the Barnett equations.

The presence of a nonzero pressure gradient in the Barnett wall sublayer induces a jump in the vapor pressure at the phase interface $P(0) > p_W$, which decreases the interphase condensation resistance. With regard to the temperature jump, the Barnett equations exhibit an analogous effect: It is slightly diminished. On the whole, however, the mass flux calculated according to the Navier–Stokes equations is greater than the value determined from the Barnett equations.

Another physical effect is associated with the exhibited inconstancy of the pressure in the Barnett sublayer: The vapor inside the sublayer is supersaturated. This effect has also been discussed in [3, 10].

The calculations carried out below apply to the case of steam at a temperature T_∞ = 303°K and a coefficient of condensation (evaporation) $\beta = 1$.

The mass fluxes calculated according to the different approximative models are compared in Fig. 2. All the curves except curves 1 and 2 are borrowed from Gajewski and others [8]. It is apparent from the figure that the mass flux in the Barnett approximation agrees well with the numerical solution based on the Bhatnagar–Gross–Krook (BGK) model in [8].

Figure 3 gives the profile of the normalized pressure $\tilde{p} = 1 - [p - p(\infty)]/[p_W - p(\infty)]$ in the Barnett sublayer for two values of the temperature difference: 1) ΔT = **2°K (curve 1); 2)** ΔT = **10°K (curve 2).**

Profiles of the normalized temperature $\tilde{T} = 1 - [T - T(\infty)]/[T_W - T(\infty)]$ and normalized density $\tilde{n} = 1 - [n - n(\infty)]/[n_W - n(\infty)]$ are given in Fig. 4 in the Barnett and Navier–Stokes approximation for two values of the temperature difference: ΔT = 2°K and 10°K. The figure reveals a characteristic density maximum in the Barnett sublayer, clearly a consequence of the variable pressure profile (it does not occur in the Navier–Stokes approximation).

The solution obtained in the present article for the slightly nonequilibrium condensation problem is more or less of a formal nature. However, the physical significance of the effects disclosed by it suggests at least the possibility of its compatibility with the concept of the Knudsen sublayer.

NOTATION

All the subscripts with the quantities ∞ are dimensioned and correspond to the state of the vapor at a large distance from the liquid phase; all others are dimensionless;

b_1	is the constant of the molecular model;
c_∞	is the average thermal velocity of molecules;
$\bar{c}(c_x, c_y, c_z)$	is the molecular velocity vector;
f	is the Barnett distribution function [1];
m	is the molecular mass;
M	is the mass flux toward the surface;

n and n_∞	are the numerical densities;
$\tilde{n} = 1-[n-n(\infty)]/[n_W-n(\infty)]$	is the normalized density;
p and p_∞	are the pressures;
$p_S(T)$	is the saturated vapor pressure at temperature T;
$p_W = p_S(T_W)$	is the saturated vapor pressure on the surface;
$\tilde{p} = 1-[p-p(\infty)]/[p_W-p(\infty)]$	is the normalized pressure;
P	is the perturbed pressure in the Barnett sublayer;
T and T_∞	are the temperatures;
T_W	is the temperature of liquid surface;
$\tilde{T} = 1-[T-T(\infty)]/[T_W-T(\infty)]$	is the normalized temperature;
v	is the transverse component of mass flow rate, $V = v/T$.
y	is the transverse coordinate;
η	is the new independent variable (2);
λ_∞	is the mean free path of molecules;
τ	is the perturbed temperature in the Barnett sublayer;
$\Delta p = p(\infty) - p_W$	are small parameters of the problem;
$\Delta T = T_\infty[T(\infty)-T_W]K.$	

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